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SUMMARY

This paper establishes that because of the possibility of drift wave propagation in the plasma in the presence of an inhomogeneity across the magnetic field, as the time goes on, the inclination of these waves' leading front becomes steeper and steeper by comparison with the initial state, so long as the mean wave frequency does not become so high that it becomes indispensable to take into account the inertia of ions across the magnetic field. Then the wave assumes a stationary state provided the amplitude is not too great. This may lead to strong plasma turbulization in the presence of instability relative to low-frequency drift waves, inasmuch as the correlation between oscillations in any two points of the plasma weakens considerably as the amplitude of oscillations rises.

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It has been established in [1, 2] that in the presence of an inhomogeneity in a plasma, transverse relative to the magnetic field, the propapation of drift waves is possible.

Assuming the plasma system at rest, let us direct the axis z along the magnetic field and the axis x along the gradient of particle density. For the sake of simplicity we postulate that the temperature of ions is zero. When plasma pressure is low, the fluctuation of the electric field may be considered as potential, and for a low frequency of oscillations the hydrodynamic velocity of ions across the magnetic field has a drift character:

$$v_{\perp} = \frac{e}{m\Omega^2}[\vec{\Omega}\vec{\nabla}\varphi], \qquad (1)$$

where $\vec{\Omega}$ is the vector of the ion-cyclotron frequency and ϕ is the electric potential.

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Taking into account that the divergence from (1) is zero, we shall write the continuity equation for ions in the form

$$\frac{\partial}{\partial t}\ln n_i + (\mathbf{v}\vec{\nabla})\ln n_i + \frac{\partial v_z}{\partial z} = 0. \tag{2}$$

Considering the plasma as quasi-neutral, we may substitute in (2) the density of electrons in place of that of ions, n_i. For a large magnetic field the components of the hydrodynamic velocity of electrons across the magnetic field may be considered as zero, and the distribution of electron density n_e as a function of potential ϕ may be taken as Boltzmann's :

$$n_e = n_0(x)\exp\{e\varphi / T\}, \tag{3}$$

where $n_0(x)$ is the mean density and T is the temperature of electrons. Substituting (3) into (2), we shall obtain, taking into account (1):

$$\begin{gathered}\frac{\partial\psi}{\partial t} + v_{\text{др}}\frac{\partial\psi}{\partial y} + v_z\frac{\partial\psi}{\partial z} + c_s^2\frac{\partial v_z}{\partial z} = 0,\\ \psi = \frac{e\varphi}{m};\quad c_s^2 = \frac{T}{m};\quad v_{\text{др}} = -\frac{c_s^2}{\Omega}\frac{\partial}{\partial x}\ln n_0.\end{gathered} \tag{4}$$

The velocity of ions along z satisfies the equation

$$\partial v_z / \partial t + (\mathbf{v}\vec{\nabla})v_z = -\partial\psi / \partial z. \tag{5}$$

Considering the drift velocity $v_{\text{др}}$ as constant, which is correct when the wavelength across the magnetic field is much less than the characteristic dimension of plasma inhomogeneity, we shall seek the solution of the system (4), (5) in the form of a simple unidimensional Reeman wave [3], i. e., we postulate

$$v_z = v_z(\psi);\quad \psi = \psi(t, \xi);\quad \xi = \mathbf{kr} = k_x x + k_y y + k_z z. \tag{6}$$

Substituting (6) into (4), (5), we shall obtain, taking into account (1):

$$\partial\psi / \partial t + (k_y v_{\text{др}} + k_z v_z + k_z c_s^2 \partial v_z / \partial\psi)\partial\psi / \partial\xi = 0; \tag{7}$$

$$\frac{\partial v_z}{\partial\psi}\frac{\partial\psi}{\partial t} + \left(k_z v_z \frac{\partial v_z}{\partial\psi} + k_z\right)\frac{\partial\psi}{\partial\xi} = 0. \tag{8}$$

Dividing (7), (8) by $\partial\psi / \partial\xi$ and denoting

$$\frac{\partial\psi}{\partial t}\Big/\frac{\partial\psi}{\partial\xi} = -\frac{\partial\xi}{\partial t}\Big|_{\psi,\, v_z} = -V, \tag{9}$$

we shall obtain

$$V = k_y v_{\text{др}} + k_z v_z + k_z c_s^2 \partial v_z / \partial\psi; \tag{10}$$

$$\frac{\partial v_z}{\partial\psi}V = k_z + k_z v_z \frac{\partial v_z}{\partial\psi}. \tag{11}$$

From (10) and (11) we eliminate $\partial v_z / \partial \psi$ and obtain

$$V = \omega_{\text{др}} + k_z v_z, \tag{12}$$

where $\omega_{\text{др}}$ is the frequency of linear drift oscillations with wave vector $\vec{k}$

$$\omega_{\text{др}} = {}^1/_2 k_y v_{\text{др}} \pm \sqrt{{}^1/_4 k_y^2 v_{\text{др}}^2 + c_s^2 k_z^2}. \tag{13}$$

Substituting (9) into (12) and integrating, we obtain

$$\xi \equiv \mathbf{kr} = \omega_{\text{др}} t + k_z v_z t + f(v_z), \tag{14}$$

where $\underline{f}$ is an arbitrary function giving the distribution of ion velocity at the moment of time t = 0. For example, if at the initial moment of time the wave was sinusoidal with amplitude v_0, we would have $f = \arcsin(v_z / v_0)$. The expression (14) defines implicitly v_z as a function of time and coordinates. Eliminating V from (10), (11), and integrating, we find that ψ is proportional to v_z.

Let us examine how the solution (14) behaves with time. It follows from (12) that at the given point V the phase velocity of the wave is not constant and depends on v_z. At those points, where v_z is greater, the wave moves faster and catches the regions where v_z is small. The nonidentity of the shift velocity of points of wave's profile leads to the variation of its shape with time [3]. The leading wave front becomes steeper and steeper, so long as for the time $t \sim 1/k_z v_0$ function $v_z(t, \xi)$ does not become discontinuous with respect to both variables, passing into a shock wave. As may be seen from (1) $v_\perp$ is proportional to the derivative from ψ, and ψ, as already noted, is proportional to v_z. This is why $v_\perp(t,\xi)$ approaches the discontinuous function much more rapidly than v_z.

In reality, the shock wave is not forming on account of the following causes. If the temperature of ions is sufficiently high, the rate of wave energy absorption increases as the steepness of the wave front rises because of ion viscosity and Landau damping on ions. As a result the wave will be absorbed prior to formation of discontinuity. But if the temperature of ions is low, the mean wave's time frequency may become so great on account of wave front's steepness increase that the accounting of ion inertia becomes indispensable.

We shall demonstrate that at frequencies, when the inertia of ions cannot be neglected and at least at small oscillation amplitudes, the existance of a stationary periodic wave in an inhomogenous plasma is possible. We derive therefrom the conclusion that in the course of time the drift wave transforms into such a wave or a superimposition of such waves.

The inertial motion of ions will be taken into account only in the first nonvanishing approximation [1]. Then instead of (1) we shall have

$$\mathbf{v}_\perp = \frac{1}{\Omega^2}\left([\vec{\Omega}\vec{\nabla}\psi] + \vec{\nabla}_\perp \frac{\partial \psi}{\partial t}\right), \tag{15}$$

and from the continuity equation and (3), (15), instead of (4) we shall have

$$\frac{\partial\psi}{\partial t} + v_{\text{дp}}\frac{\partial\psi}{\partial y} + \frac{c_s^2}{\Omega^2}\Delta_\perp \frac{\partial\psi}{\partial t} + v_z\frac{\partial\psi}{\partial z} + c_s^2\frac{\partial v_z}{\partial z} = 0. \tag{16}$$

Since the inertia of ions is taken into account only in oscillation amplitude-wise terms, we may substitute into (16) the expression [see (1), (5), (6)]

$$\partial\psi / \partial z = -\partial v_z / \partial t - v_z \partial v_z / \partial z. \tag{17}$$

We shall seek the solution of the system (16), (17) in the form of a stationary wave, that is, we postulate

$$\psi = \psi(\xi_1); \quad v_z = v_z(\xi_1); \quad \xi_1 = \mathbf{kr} - \omega t. \tag{18}$$

Then from (16), (17)

$$\left(\frac{c_s k_\perp \omega}{\Omega}\right)^2 \frac{\partial^2}{\partial\xi_1^2} v_z + (\omega^2 - \omega k_y v_{\text{дp}} - k_z^2 c_s^2)\, v_z - {}^1\!/_2 k_z(\omega - k_y v_{\text{дp}})\, v_z^2 + {}^1\!/_3 k_z^2 v_z^3 = 0. \tag{19}$$

For great ω (19) has periodical solutions, for then it is the equation of a nonlinear oscillator. We see that without the first term, containing the small parameter Ω^{-2} and the second derivative from v_z, the drift oscillations have no stationary oscillating solutions (in accord with (14)). As follows from (14), the derivatives with respect to time and coordinates from v_z rise and the first term of (19) becomes material. Then, the subsequent drift wave "changeover" ceases and the drift wave transforms into a superimposition of periodical solutions of (19). The superposition of periodic waves gives the quasistationary wave described by Eqs.(16), (17).

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